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Generalized Uncertainty Principle in a Simple Varying Speed of Light Model

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In this paper we will derive a generalized uncertainty principle (GUP) in a simple varying speed of light (VSL) model. First we will show that VSL is an immediate consequence of GUP. Then, within the framework of a simple VSL model, we will show that GUP can be expressed as a function of cosmological scale factor. This expression gives two main results: uncertainties in position and momentum are actually cosmological models dependent and these uncertainties depend on mass and momentum of the particle under consideration. The relationship between matter content of the Universe and the values of uncertainties in early stages of the evolution of the Universe will be discussed in a mini-superspace approach.

KEY WORDS: quantum gravity; generalized uncertainty principle; varying speed of light models.

1. INTRODUCTION

In recent years it has been suggested that measurements in quantum gravity are governed by generalized uncertainty principle. In fact some evidence from string theory and black holes physics, based on gedanken experiments (Amati *et al.*, 1987, 1988, 1989, 1990; Garay, 1995; Gross and Mende, 1987, 1988; Guida *et al.*, 1991; Konishi *et al.*, 1990; Veneziano, 1986b), leads some authors to re-examine usual uncertainty principle of Heisenberg (Castro, 1995; Camacho, 2002; Capozziello *et al.*, 1999; Chen, 2003; Maggiore, 1993a). These evidences have origin on the quantum fluctuations of the background spacetime metric. Introducing of this idea has drown attention and many authors have considered various problems in the framework of generalized uncertainty principle. Maggiore by considering GUP in quantum gravity has indicated that a minimum length of the

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order of the Planck length emerges naturally from any theory of quantum gravity (Maggiore, 1993a, 1994). He has investigated also the relationship between the GUP in quantum gravity and quantum deformation of the Poincaré algebra and has shown that in the deformed Poincaré algebra a minimal observable length emerges naturally (Maggiore, 1993b). He has obtained the algebraic structure of GUP and has reproduced the string theory result for GUP (Maggiore, 1993a, 1993b, 1994). Castro has derived the modifications of Heisenberg uncertainty principle by considering the problem in the framework of the theory of Special Scale-Relativity (Castro, 1995). Capozziello and his coworkers by considering the existence of an upper limit on the acceleration of massive particles, have derived the GUP of string theory in the framework of quantum geometry (Capozziello *et al.*, 1999). Adler and his coworkers have argued that contrary to standard viewpoint, GUP may prevent small black holes total evaporation in exactly the same way as the uncertainty principle prevents the Hydrogen atoms from total collapse (Adler *et al.*, 2001). Kalyana Rama has studied dynamical consequences of GUP. He has argued that GUP can lead naturally to varying speed of light (VSL) and modified dispersion relations (Kalyana Rama, 2001). Camacho has analyzed the role which GUP could play in quantization of the electromagnetic field (Camacho, 2003b). Recently various problems have been considered in the framework of GUP. The problem of dark matter as has been indicated by Adler and his coworkers (Chen and Adler, 2002) has been considered in more details by Chen. He has argued that dark matter might be composed of Planck-size black holes remnants (Chen, 2003). Scardigli and Casadio have investigated the possibility of the existence of extra dimensions in the framework of GUP (Scardigli and Casadio, 2003). Time evolution of a quantum particle in a homogeneous gravitational field in the framework of GUP is considered by Camacho (2003a) and quantum field theoretical view point to GUP and its consequences is considered by Kempf *et al.* (1995, 1996, 1997).

Recently, varying speed of light, as a new conjecture, which has been proposed to solve the problems of standard cosmology, has attracted most of the attentions. After introducing this conjecture, several alternative VSL theories have been proposed and some of their novel implications have been examined (Albrecht, 1999; Magueijo, 2000, 2001; Barrow and Magueijo, 1998; Barrow, 1999). Along such investigations, in this paper first we will show that VSL is a consequence of GUP. Then considering combination of GUP and a simple varying speed of light model, we will show that GUP is actually dependent to the type of cosmological model under consideration via scale factor. The dependence of GUP to cosmological scale factor, permits one to consider the problem in model Universes with different matter content within some considerations of VSL theories. A mini-superspace approach for universe as a free particle is presented to compute the product of uncertainties in position and momentum and the results are explained.

The structure of the paper is as follow: Section 2 is devoted to the Commutators algebra to prepare the basics of the calculations. Section 3 considers Hamiltonian for generalized commutative relations to incorporate dynamics. Section 4 considers the GUP in a simple VSL model. Section 5 is devoted to a mini-superspace approach to compute the product of uncertainties in position and momentum. The paper follows by remarks and conclusions in Section 6.

2. COMMUTATORS ALGEBRA

Usual uncertainty principle of quantum mechanics, the so-called Heisenberg uncertainty principle, should be re-formulated because of the non-commutative nature of spacetime (Gibbons and Hawking, 1977; Wheeler, 1957). It has been indicated that in quantum gravity there exists a minimal observable distance on the order of the Planck length. In the context of string theories, this observable distance is referred to GUP (Amati *et al.*, 1987, 1988, 1989, 1990; Ciafaloni, 1992; Gibbons and Hawking, 1977; Veneziano, 1986a; Gross and Mende, 1987; Kato, 1990; Konishi *et al.*, 1990; Wheeler, 1957):

$$\Delta x \geq \frac{\hbar}{\Delta p} + \text{const. } G\Delta p. \tag{1}$$

At energies much below the Planck mass, $M_{\rm Pl}$, the extra term in Eq. (1) is negligible and one recover the usual Heisenberg uncertainty principle, but as one approaches the Planck mass, this term becomes important and it is responsible for existence of a minimal observable distance as mentioned above.

One can construct a κ-deformed algebra and show that the most general form of this algebra is (Maggiore, 1993a, 1993b, 1994)

$$[x_i, x_j] = \frac{\hbar^2 a(E)}{\kappa^2} i\epsilon_{ijk} J_k \tag{2}$$

$$[x_i, p_j] = \quad i\hbar\delta_{ij} f(E). \tag{3}$$

where $a(E)$ and $f(E)$ are real, dimensionless function of $\frac{E}{\kappa}$ and $E^2 = p^2c^2 + m^2c^4$. The angular momentum $\mathbf{J}$ is defined as dimensionless quantity, $j_i = -i\epsilon_{ijk} p_j \frac{\partial}{\partial p_k}$. Since the generalized uncertainty principle will be derived from relations (2) and (3), one should compute the explicit form of $a(E)$ and $f(E)$.

We start from Jacobi identities $[x_i, [x_j, x_k]] + \text{cyclic permutations} = 0$ to obtain $[x_i, E] = i\hbar p_i \frac{f(E)}{E}$ and using $[x_i, G(\vec{p})] = i\hbar \frac{\partial G}{\partial p_i}$ one can show that $[x_i, a(E)] = i\hbar p_i \frac{f(E)}{E} \frac{\partial a}{\partial E}$. Now some algebra gives,

$$\frac{\partial a}{\partial E} \vec{P}.\vec{J} = 0. \tag{4}$$

Since the Jacobi identities are satisfied independently of the particular representation of the algebra, that is independently of whetherthe condition

$\vec{P}.\vec{J} = 0$ holds or not, one conclude that $\frac{\partial a}{\partial E} = 0$ and therefore arrive at the result $a(E) = \text{const}$. With a redefinition of κ we can set this constant to ± 1. For determining $f(E)$, one must repeat the above lines once again. The Jacobi identity $[x_i, [x_j, p_k] + \text{cyclic permutations} = 0$ gives the following result,

$$\frac{f(E)}{E}\frac{\partial f}{\partial E} = \mp\frac{1}{\kappa^2} \tag{5}$$

where the upper (lower) sign corresponds to the choosing $a = +1(-1)$, respectively. In order to recover Heisenberg uncertainty principle one should consider $f(0) = 1$, and integration of the Eq. (5) gives (the solution with the upper sign is however quite intriguing; since $f(E)$ is real by definition, the solution is valid only for $E \leq \kappa$ and describe a system which obeys standard quantum mechanics for $E \ll \kappa$ and satisfies$[x, p] = 0$ at $E = \kappa$)

$$f(E) = \sqrt{1 + \left(\frac{E^2}{\kappa^2}\right)}. \tag{6}$$

In which follows we write the κ-deformed Heisenberg algebra as

$$[x_i, x_j] = -\epsilon\frac{\hbar^2}{\kappa^2}i\epsilon_{ijk}J_k \tag{7}$$

$$[x_i, p_j] = i\hbar\delta_{ij}\sqrt{1 + \frac{E^2}{\kappa^2}} \tag{8}$$

where $\epsilon = \pm 1$. Generally one can write generalized uncertainty principle as the following form,

$$\triangle x_i \triangle p_j \geq \frac{\hbar}{2}\delta_{ij}\langle f\rangle. \tag{9}$$

Therefore, from Eq. (8) we derive the generalized uncertainty principle as

$$\triangle x_i \triangle p_j \geq \frac{\hbar}{2}\delta_{ij}\left\langle\sqrt{1 + \frac{E^2}{\kappa^2}}\right\rangle. \tag{10}$$

Now assuming $E^2 \ll \kappa^2$, by expanding the square root in powers of $\left(\frac{E}{\kappa}\right)^2$ and using $\langle p^2\rangle = p^2 + (\triangle p)^2$, where $(\triangle p)^2 = \langle(p - \langle p\rangle)^2\rangle$, the first order term in expansion of Eq. (10) is,

$$\triangle x_i \triangle p_j \geq \frac{\hbar}{2}\delta_{ij}\left(1 + \frac{E^2 + (\triangle p)^2}{8\kappa^2}\right). \tag{11}$$

It is important to note that Eq. (11) gives different results in different limits. For example if one take $E \ll \kappa$ and $\triangle p \leq \kappa$, this equation leads to the result of string theory, i.e., Eq. (1).

Since commutation relations are kinematical, for the rest of dynamics we should determine Hamiltonian. The next section considers dynamics and a suitable form for f as has been appeared in Eq. (9).

3. HAMILTONIAN FOR GENERALIZED COMMUTATIVE RELATIONS

Now we consider the consequences of the generalized commutation relations (GCRs) in order to derive suitable Hamiltonian. It is important to note that the commutation relations are only kinematical quantities. The dynamics is determined by Hamiltonian H, which need to be specified. For simplicity in which follows we assume that H depends only to P and this is the case for free particle. In this case, H is independent of position, X.

Two choices for H can be considered,

$$H = \sqrt{P^2c^2 + m^2c^4} \tag{12}$$

and

$$\sinh \lambda H = \lambda\sqrt{P^2c^2 + m^2c^4}. \tag{13}$$

In general c can be a function of space coordinates and time (Albrecht, 1999), but we will consider only time dependence of c in which follows. Equation (12) is the usual Hamiltonian for free particles, while H in (13) as has been indicated in (Maggiore, 1993a), is the first Casimir operator.

We define the velocity, V_i, by

$$V_i \equiv \frac{dX_i}{dt} = \frac{i}{\hbar}[H, X_i]. \tag{14}$$

Here we consider only the non-rotating systems, and for such systems, $[X_i, X_j] = 0$, and then using (3) we obtain,

$$V_i = \frac{fH'P_ic}{\sqrt{P^2c^2 + m^2c^4}} \tag{15}$$

where H' is the derivative of H with respect to $\sqrt{P^2c^2 + m^2c^4}$. The eigenvalue of V_i are v_i's and the speed v of a particle with mass m is

$$v = \sqrt{\sum_{i=1}^{d} v_i^2} = \frac{fE'pc}{\sqrt{p^2c^2 + m^2c^4}} \quad \text{and} \quad v \leq c, \tag{16}$$

which is the modified dispersion relation. It is evident that the speed of light is the speed of a massless particle. Equation (16) then gives (Kalyana Rama, 2001)

$$c = fE'. \tag{17}$$

In Eqs. (16) and (17), p and f must be expressed in terms of E, since an explicit form of H is required. For H given by (12), after integration one find,

$$v = \frac{\sqrt{(E^2 - m^2c^4)(1 + \lambda^2 E^2)}}{E} \quad \text{and} \quad c = \sqrt{1 + \lambda^2 E^2}. \tag{18}$$

As it is clear from Eq. (17), generically c is varying as a function of E. Such VSL theories have been extensively studied (Albrecht, 1999; Magueijo, 2000, 2001; Barrow and Magueijo, 1998; Barrow, 1999), and found to have non trivial implications for cosmology (Barrow and Magueijo, 1998) and black hole physics (Magueijo, 2001). So the most important aspect of the GUP is that these relations lead to VSL. Note that actually VSL theories consider c varying as a function of cosmic time or scale factor. Here we have shown that c is a function of E. The question whether c is varying with time or energy is not important, the important matter here is the possibility of variation in c and as we have shown, c is varying.

4. GENERALIZED UNCERTAINTY PRINCIPLE IN A VARYING SPEED OF LIGHT MODEL

As has been indicated in the last section, c can be varying. Now, instead of considering variation of c with energy, we consider c to be varying as a function of time since any variation in energy is related to variation in time (remember that Hamiltonian is the generator of time evolution). By this assumption, we obtain a generalized uncertainty principle (GUP) that depends on cosmological scale factor. We define a parameter $\lambda = \frac{\hbar c}{\kappa}$ which has length dimension. Accordingly, Eq. (6) now takes the following form,

$$f = \sqrt{1 + \frac{\lambda^2}{\hbar^2}(p^2 + m^2c^2)}, \tag{19}$$

where $E^2 = p^2c^2 + m^2c^4$. We will consider c as a function of time as has been proposed by Barrow (1999). As Barrow has shown, one can consider the speed of light as a function of scale factor,

$$c(t) = c_0 a^n(t) \tag{20}$$

where c_0 is a constant, a is scale factor and n is a constant which depends on the nature of the solutions and matter content of the Universe and impose some constraints to solve horizon, flatness and others problems of standard cosmology.

The existence of a minimal length which is of the order of Planck length leads to the new commutator for x and p as (Kempf *et al.*, 1995, 1997),

$$[x, p] = i\hbar(1 + \beta p^2), \tag{21}$$

where β is a function of Planck length. So it is reasonable to consider λ proportional to Planck length. Since $l_{\rm Pl} = (\frac{G\hbar}{c^3})^{1/2}$, we find,

$$f = \sqrt{1 + \frac{l_{\rm Pl}^2}{\hbar^2}(p^2 + m^2c^2)}, \tag{22}$$

or

$$f = \sqrt{1 + \frac{G}{\hbar c^3}(p^2 + m^2c^2)}. \tag{23}$$

Using (21), we find,

$$f = \sqrt{1 + \frac{G}{\hbar(c_0 a^n(t))^3}(p^2 + m^2(c_0 a^n(t))^2)}, \tag{24}$$

Now using this result in Eq. (9), generalized uncertainty principle can be written as,

$$\triangle x_i \triangle p_j \geq \frac{1}{2}\delta_{ij}\hbar \left\langle \sqrt{1 + \frac{G}{\hbar(c_0 a^n(t))^3}(p^2 + m^2(c_0 a^n(t))^2)} \right\rangle. \tag{25}$$

This relation clearly shows that generalized uncertainty principles (GUP) are explicit functions of scale factor and therefore GUP depends on cosmological model under consideration. Since a cosmological model depends on the matter content of the Universe, GUP can take different form in different cosmological models. An other implication of this result is the dependence of uncertainties in x and p on mass and momentum of the particle. This is in complete agreement with the results of Camacho (2003b) and Kempf *et al*., (1995).

Suppose that the matter content of the Universe at its early stage of the evolution, is given by a cosmological constant. Suppose the equation of state being $P = w\rho c^2(t)$. Since for this type of matter content, $w = -1$, by analysis which has been done in (Barrow, 1999), one can restrict n to the $n \leq 1$. So we can choose as a possible alternative, $n = 1$. In this situation, Eq. (25) gives the following result,

$$\triangle x_i \triangle p_j \geq \frac{1}{2}\delta_{ij}\hbar \left\langle \sqrt{1 + \frac{G}{\hbar(c_0 a(t))^3}(p^2 + m^2(c_0 a(t))^2)} \right\rangle. \tag{26}$$

If we set df/da equal to zero we get the result $a(t) = 0$. With this value of $a(t)$, $f(t)$ becomes infinite. This is not surprising because with cosmological constant as matter content of the Universe, the Universe undergoes to an inflationary phase transition and in this situation uncertainties can be very large. One can compute the expectation value using the wave function for a free particle but the result is completely cosmological model dependent. As another example, consider a model universe with radiation as matter content. Now $w = \frac{1}{3}$ and $n < -2$. We choose n to be $-\frac{5}{2}$ and so from equation (25), one find,

$$\triangle x_i \triangle p_j \geq \frac{1}{2}\delta_{ij}\hbar \left\langle \sqrt{1 + \frac{G}{\hbar(c_0 a^{-5/2}(t))^3}(p^2 + m^2(c_0 a^{-5/2}(t))^2)} \right\rangle. \qquad (27)$$

The same procedure as the last paragraph leads to the result $a(t) = 0$ and therefore from Eq. (27) one find

$$\triangle x_i \triangle p_j \geq \frac{1}{2}\delta_{ij}\hbar \qquad (28)$$

which is the usual Heisenberg uncertainty relation. These results show that actually we encounter large uncertainty in early stages of the Universe because of successive phase transitions and quantum fluctuation of background metric and these uncertainties increase with expansion of the universe (except for oscillating Universe!).

5. MINI-SUPERSPACE CONSIDERATIONS

To have an explicit form for the values of uncertainties, one should specify $\langle f \rangle$ in Eq. (25). To do so, one should consider a specific wave function to compute $\langle f \rangle$. Consider, for example, the early Universe. Suppose that the early Universe can be considered as a free particle. Then in the context of Hartle–Hawking mini-superspace approach, the expectation value for f in this state is

$$\langle f(t) \rangle = \langle \Psi^{HH} | f(t) | \Psi^{HH} \rangle, \qquad (29)$$

and since Hartle–Hawking wave function for Universe in no-boundary proposal and mini-superspace approach is (Kolb and Turner, 1990)

$$\Psi^{HH} = N \int_0^\infty \cos\left(\frac{y^3}{3} + \left(\frac{3\pi a_0^2}{4G}\right)^{\frac{2}{3}} \left(1 - \frac{a^2(t)}{a_0^2}\right) y\right) dy. \qquad (30)$$

one find the following integral,

$$\langle f(t) \rangle = N^2 \int_0^\tau \left[\int_0^\infty \cos\left(\frac{y^3}{3} + \left(\frac{3\pi a_0^2}{4G}\right)^{\frac{2}{3}} \left(1 - \frac{a^2(t)}{a_0^2}\right) y\right) dy\right]^2$$

$$\times \sqrt{1 + \frac{G}{\hbar(c_0 a^n(t))^3}(p^2 + m^2(c_0 a^n(t))^2)}\, dt, \qquad (31)$$

which can not be solved analytically. Now, for three possible candidates of the Universe evolution,

$$a(t) = a_0 \cosh\left(\frac{t}{a_0}\right) \qquad \text{De Sitter Universe,} \tag{32}$$

$$a(t) = a_0 \exp(H_0 t) \qquad \text{Inflationary Universe,} \tag{33}$$

$$a(t) = a_0 \cos\left(\frac{t}{a_0}\right) \qquad \text{Oscillating Universe,} \tag{34}$$

we have computed Eq. (31) numerically and the results are shown in Fig. 1. For simplicity, we have assumed in the numeric calculation that, $a_0 = G = \hbar = c_0 = \tau = H_0 = N = m = p = \text{const} = 1$ and $n = -1$. As figure shows, in early stages of the evolution, product of uncertainties behaves oscillatory because of successive phase transitions and then after, by expansion of the universe, as usual quantum mechanics, uncertainties increase. This increasing uncertainties has simple explanation of broadening of the wave function of the universe. As is evident from figure, in an oscillating Universe, the product of uncertainties decreases. But this

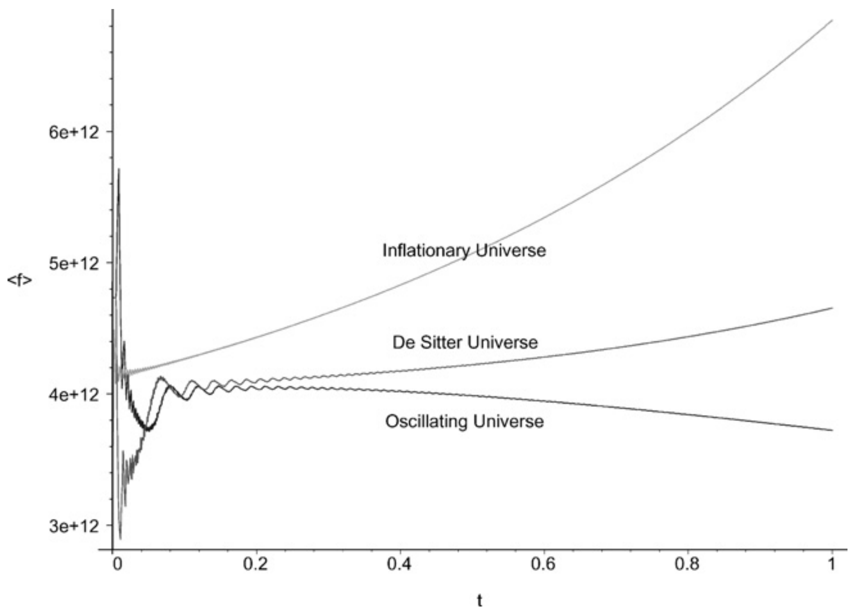

Fig. 1. Variation of product of uncertainties for three candidate models of the universe in a mini-superspace approach.

reduction continue only till approaching big crunch and then after this situation will be repeated.

6. REMARKS AND CONCLUSIONS

As has been shown, generalized uncertainty principle can be considered in the framework of varying speed of light models. Actually variation of the light speed is an immediate consequence of the generalized uncertainty principle. The result of joining these two novel aspects of modern cosmology, is the fact that generalized uncertainty principle is a function of scale factor and consequently in various cosmological models it takes different form. Our analysis shows that cosmological models with different matter content and therefore with different scale factor give different uncertainties. Our analysis shows that in the early stages of the evolution of the Universe the value of uncertainties was considerable and this has origin in successive phase transitions and quantum fluctuation of background spacetime metric. Mini-superspace considerations compare three possible candidate for the early Universe and as figure shows they have different values of uncertainties.

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